

Weakening termination conditions, via W.H.J. Feijen

The purpose of this short note is to generalize the little ‘trick’ Wim Feijen presented in WF153 - ‘To my fellow teachers of programming’.

We are given N , $0 \leq N$ and array $f[0..N)$. Let \oplus be a symmetric and associative binary operator with identity a and zero e . We define K as follows:

$$K.n = \langle \oplus i : n \leq i < N : f.i \rangle$$

With postcondition R and invariant P ,

$$R : K.0 = x$$

$$P : K.0 = x \oplus K.n$$

we may choose for the termination condition $x = x \oplus K.n$. We calculate

$$\begin{aligned} & x = x \oplus K.n \\ \Leftarrow & \quad \{ \text{either } x \text{ is the zero of } \oplus \text{ or } K.n \text{ is the identity of } \oplus \} \\ & x = e \vee K.n = a \\ \Leftarrow & \quad \{ \text{property of } K; \text{ empty range} \} \\ & x = e \vee n = N \end{aligned}$$

and thus we have derived our termination condition for many familiar operators like \forall , \exists , Σ , Π etc. in one go.

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