Weakening termination conditions, via W.H.J. Feijen

The purpose of this short note is to generalize the little 'trick' Wim Feijen presented in WF153 - 'To my fellow teachers of programming'.

We are given N, $0 \leq N$ and array $f[0..N)$. Let $\oplus$ be a symmetric and associative binary operator with identity a and zero e. We define K as follows:

$$K.n \; = \; \langle \oplus i : \; n \leq i < N : \; f.i \rangle$$

With postcondition R and invariant P,

$R: \quad K.0 \; = \; x$

$P: \quad K.0 \; = \; x \oplus K.n$

we may choose for the termination condition $x \; = \; x \oplus K.n$. We calculate

$$\begin{array}{ll} & x \; = \; x \oplus K.n \\ \Leftarrow & \quad \{\text{either } x \text{ is the zero of } \oplus \text{ or } K.n \text{ is the identity of } \oplus\} \\ & x \; = \; e \;\; \vee \;\; K.n \; = \; a \\ \Leftarrow & \quad \{\text{property of } K\text{; empty range}\} \\ & x \; = \; e \;\; \vee \;\; n \; = \; N \end{array}$$

and thus we have derived our termination condition for many familiar operators like $\forall$, $\exists$, Σ, Π etc. in one go.

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