6.

Prove that if Eqs. (6) hold before step E4 is performed, they hold afterwards also.

* * *

We are given

$$A5: \quad a*m \ + \ b*n = d \quad \wedge \quad a'*m \ + \ b'*n = c \quad \wedge \quad c = q*d \ + \ r \quad \wedge \quad 0 < r \quad \wedge \quad r < d \quad \wedge \quad c \downarrow d \ = \ m \downarrow n$$

$$A6: \quad a*m \ + \ b*n = d \quad \wedge \quad a'*m \ + \ b'*n = c \quad \wedge \quad d > 0 \quad \wedge \quad c \downarrow d \ = \ m \downarrow n$$

$$(6) \quad a*m \ + \ b*n = d \quad \wedge \quad a'*m \ + \ b'*n = c$$

$$(*) \quad \{A5\} \ E4 \ \{A6\}$$

where $x \downarrow y$ is the gcd of x and y.

We are asked to show

$$(**) \quad \{(6)\} \ E4 \ \{(6)\}$$

We can do so by showing that $A5 \Rightarrow (6) \quad \wedge \quad A6 \Rightarrow (6)$. We observe

$$\begin{array}{ll} & A5 \\ = & \quad \{\text{by definition}\} \\ & a*m + b*n = d \quad \wedge \quad a'*m + b'*n = c \quad \wedge \quad c = q*d + r \quad \wedge \quad 0 < r \quad \wedge \quad r < d \quad \wedge \quad c \downarrow d \ = \ m \downarrow n \\ \Rightarrow & \{X \wedge Y \quad \Rightarrow \quad X\} \\ & a*m \ + \ b*n = d \quad \wedge \quad a'*m \ + \ b'*n = c \\ = & \quad \{\text{by definition}\} \\ & (6) \end{array}$$

By a similar argument we can conclude $A6 \Rightarrow (6)$.

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