Throughout my life, I have pursued almost every area of interest I have encountered, from literature and music, to linguistics and mathematics, to backgammon and cryptic crosswords, to basketball and frisbee. The common thread in these pursuits is the need for creativity and discipline: Creativity helps us design new problems and techniques, while discipline helps us solve the problems efficiently, and exploit the techniques effectively. These skills are however quite difficult to master, and my primary interest is in exploring methodologies of teaching them. In particular, I research the field of calculational mathematics, which I feel provides the ideal environment for developing creativity and discipline. Through graduate study, I will create an exposition of this calculational style, so it can be explored by a wider audience, both mathematical and general. Ultimately, my research will lead to a book on pedagogy, and the foundation of a school.

Creativity is difficult to master because it is vague and open-ended. We may not have a clear idea of what we want to create, and even when we do we may have no idea where to begin. So to apply our creative faculties more effectively, we need the ability to clearly define our goals, to break problems into smaller pieces, to be opportunistic. And to build upon what we know, we need --alongside a rich foundation of knowledge-- the ability to abstract from details, to form analogies and generalizations.

Discipline of thought is difficult to master because it conflicts with natural, subconscious thought. The subconscious techniques of habit, intuition, and trial-and-error stand in opposition to the conscious goals of being organized and explicit, identifying hidden assumptions, avoiding needless distinctions, as well as the principles of creativity above. Though these principles are easily stated, they are difficult to apply well, because the mind so easily slips into the natural mode.

Awareness of these principles has not only made me a better scientist, but a better artist as well. For example, I often write poetry by first choosing my themes, then finding phonetic sounds which I feel evoke them, then developing a rhythmic scheme for organizing the sounds, and finally choosing words to implement them. I compose cryptic crossword clues similarly, creating new words from the puzzle's letters, using wordplay. And when I practice the piano, I pay attention to the patterns my hands and arms fall into, the notes or passages I continually miss, and then work consciously to correct these patterns through discipline.

Like any skill, creativity and discipline must be developed through practice, and mathematics is ideal for this purpose. When we are just learning to think, the concepts we reason about should be very simple, so we can focus on the reasoning itself. Learning to think by trying to solve the problem of world hunger is like learning to swim in a tidal wave -- you will be drowned in complexity. So, even though the real-world concepts we ultimately wish to reason about are complex and hazy, it is essential to practice in an idealized domain. And since mathematical properties are simpler and more easily articulated than nonmathematical ones, mathematical concepts are the ideal concepts to work with when learning to think.

Concepts in calculational mathematics are completely precise, meaning they can be formulated in terms of symbol manipulation. (These are the sorts of properties we use so well in computing $346 + 123$, or in solving $2x + 1 = 3$.) Because calculating is just symbolic manipulation, we can focus entirely on questions of *how* to manipulate: how to define goals clearly, separate concerns, and form generalizations; how to be explicit, organized, and aware of choices. These skills, which outside of mathematics are fairly vague, can now be concretely understood and explicitly taught in terms of symbols.

The goal of the calculational style is not to try to reduce all problems to symbol manipulation, but to use calculation as a tool wherever possible, and, more importantly, to foster the skills of creativity and discipline of thought (and a taste for simplicity) which become so crucial as the complexity of problems grows. Thus it has greatly aided my work in mathematics and computing science: I use calculational techniques to design new proofs and programs in a disciplined, orderly fashion, to clarify and streamline work in the literature, and to develop and refine methodologies of mathematics and programming.

The first phase of my research is to create a work that functions simultaneously as a reference within the calculational community, and as a crisp yet thorough explanation of our methods to the wider scientific community. Despite potential benefits of the calculational style both in scientific research and in pedagogy, it is relatively unknown in the academic world, primarily because there has never been a complete and coherent exposition of the style. Through graduate study, I intend to create such an exposition, synthesizing and extending the essential ideas and tools of our style that have developed over the past 20 years.

The second phase, to be completed after my graduate study, is to expand this work into a book on pedagogy, accessible to any reader. The first half of the book will describe in detail the aspects of creativity and discipline, and the role mathematics has to play in developing these skills, as outlined above. The second half will give readers the technical background they need to design their own mathematical arguments, including an exposition of predicate calculus, lattice theory, and algorithm design.

The third phase is to realize this pedagogical framework in my own school. At the center of the curriculum will be a class on thinking, where a student will first become aware of their own thought processes, so as to harness their mind as a tool, and then sharpen that tool through the calculational style. Along the way, they will apply their skills to a variety of subjects, from music, to literature, to science, to sport, so that their abilities of creative expression and rational thought can develop in tandem.

In her PhD thesis, “On the shape of mathematical arguments” , Dutch computing scientist Netty van Gasteren showed how the presentation of mathematical proofs can be improved by adopting syntactic, calculational techniques. Through the efforts of many computing scientists and mathematicians, these techniques have been crafted into a powerful and teachable mathematical methodology, whereby proofs can be constructively designed from their specifications, rather than intuited. Studying under Wim Feijen, van Gasteren’s long-time collaborator and mentor, I played a role in this development through my Fulbright research in the Netherlands. I wish to continue this research at the University of Nottingham with one of their closest colleagues, Roland Backhouse.

Since the calculational techniques were originally developed in response to the complexity of program design, several works have explored calculational programming methodology, for example, Roland Backhouse’s “Program Construction” and van Gasteren and Feijen’s “On a Method of Multiprogramming” , to name only two. But while the broader mathematical methodology has been applied successfully to diverse areas of mathematics, the details of the methodology itself have only been touched upon in the mathematical literature.

I aim to remedy this by conducting an in-depth investigation of the calculational style, in the process synthesizing and extending van Gasteren’s research, and later research, including my own. My work will serve three purposes. Firstly, it will fulfill a long-standing need within the calculational community, by culling together the essential ideas and tools of the calculational style. Secondly, it will increase the exposure of our methodology to the wider scientific community. And finally, because the calculational method is so explicit and teachable, it will lay the groundwork for a new pedagogy of mathematics.

For the structure of my investigation, I adapt the approach taken by van Gasteren, which has two threads. In the first thread, I discuss and develop the techniques of the modern calculational style, from the principles of naming, interface design, abstraction, and separation of concerns; to techniques of human calculation, Wim Feijen’s proof format and its refinements, predicate calculus, lattice theory, and heuristics for the exploitation of properties like transitivity. In the second thread, I apply our methodology to several non-trivial problems from the literature, showing how the calculational style not only results in cleaner expositions, but also caters to the construction of solutions by analysis and design.

Nottingham’s three-year degree program centers exclusively on a doctoral thesis, allowing me to begin my research and collaboration straight away. My advisor, Roland Backhouse, has written books and articles on the calculational style, led the mathematical programming group at the University of Eindhoven for nine years, and currently leads the Foundations of Programming Group in Nottingham. Having met him and his graduate students personally, and having corresponded with them for the last year, I am convinced that we will have a fruitful collaboration. Finally, I have many colleagues nearby in England and mainland Europe, who I met during my year in the Netherlands. In short, Nottingham will provide a highly charged environment in which to undertake my graduate research.