Exercise 10 from WF122

Next we investigate:

(0)   [$(X \equiv Y) \Rightarrow (X \Rightarrow Y)$]   .

Because I missed the opportunity in EX5 , I will make sure to try a wide calculation:

$\qquad (X \equiv Y) \Rightarrow (X \Rightarrow Y)$

$\equiv \qquad \{$ shunting $\}$

$\qquad (X \equiv Y) \wedge X \Rightarrow Y$

$\equiv \qquad \{$ Modus Ponens $\}$

$\qquad X \wedge Y \Rightarrow Y$

$\equiv \qquad \{$ weakening $\}$

   true .

Also, in EX5a we learned how to distribute $\Rightarrow$ leftward over $\equiv$:

$\qquad (X \equiv Y) \Rightarrow (X \Rightarrow Y)$

$\equiv \qquad \{ \Rightarrow$ leftward over $\equiv \}$

$\qquad \pm X \Rightarrow (X \Rightarrow Y) \equiv \mp Y \Rightarrow (X \Rightarrow Y)$

$\equiv \qquad \{ \Rightarrow$ into $\neg/\vee$ to homogenize $\}$

$\qquad \mp X \vee \neg X \vee Y \equiv \pm Y \vee \neg X \vee Y$

$\equiv \qquad \{$ with $\neg X/+Y$ both sides equivale $\neg X \vee Y$; with $+X/\neg Y$, both
       sides equivale **true** $\}$

   true .

That's not nice at all!

Perhaps the best approach is to use EX7 , which relates the booleans $X \equiv Y$ and $X \Rightarrow Y$:

EX10–1

$$X \equiv Y$$

$\equiv$ { mutual implication }

$$(X \Rightarrow Y) \ \wedge \ (Y \Rightarrow X)$$

$\Rightarrow$ { weakening }

$$X \Rightarrow Y \qquad .$$

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