**Exercise 9 from WF122**

Now that I have my pen back, I would like to design a proof of:

(0) $[\ \mathbf{true} \Rightarrow X \ \equiv \ X\ ]$ ,

which shows the viability of proving $X$ by strengthening it to **true** , a la:

$X$

$\Leftarrow$ { ... }

**true** .

As in the previous EX , we should strive to rewrite (0) to combine **true** with $\equiv$ , $\wedge$ , or $\vee$ , because we know properties relating **true** to these.

Thus I calculate:

$\mathbf{true} \Rightarrow X$

$\equiv$ { $\Rightarrow$ into $\neg/\vee$ }

$\neg\mathbf{true} \vee X$

$\equiv$ { predicate calculus }

$\mathbf{false} \vee X$

$\equiv$ { property of **false** }

$X$

and:

$\mathbf{true} \Rightarrow X$

$\equiv$ { $\Rightarrow$ into $\equiv/\wedge$ }

$\mathbf{true} \ \equiv\ \mathbf{true} \wedge X$

$\equiv$ { **true** is identity of $\equiv$ and $\wedge$ }

$X$

and:

$$
\begin{array}{ll}
 & \mathbf{true} \Rightarrow X \\
\equiv & \quad \{ \ \Rightarrow \ \text{into} \ \equiv / \vee \ \} \\
 & X \equiv X \vee \mathbf{true} \\
\equiv & \quad \{ \ X \vee \mathbf{true} \equiv \mathbf{true} \ \} \\
 & X \equiv \mathbf{true} \\
\equiv & \quad \{ \text{ predicate calculus } \} \\
 & X \qquad .
\end{array}
$$

What more is there to say?

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