Something that might make it into my book

One of the most striking aspects of the calculational style in mathematics is that it advocates manipulating symbols and formulae using strict rules, without interpreting them, without assigning them any meaning.

On the other hand, as far as I can tell, traditional mathematical education —the kind one finds through high school— stresses concepts above all. For example, when we are first taught to add 1 and 2 to get 3, we are told that what we are really doing is, for example, putting one apple together with two apples, and getting three. Addition is just an abstraction, a tool, meaningless without having something to add. The apples (or whatever) are what we are really studying, and we are encouraged at every point to not lose ourselves in mathematics, but always remember what we are really doing.

Similarly, notation is traditionally used only as a means of succinctly capturing concepts so they can be transmitted to others. For example, we are taught that the addition problem above can be expressed by the equation $1+2=3$, but at the same time that this equation is just yet another abstraction of the mathematical concept of addition, which, recall, is itself only an abstraction of the "real-world" concept of putting some stuff together to get more stuff. This abstraction is redeemed only by the fact that $1+2=3$ is pretty easy to write, and we are encouraged at every point not to lose ourselves in notation, but always remember what the notation stands for. We are only capable of really understanding the concepts; after all, we are human beings, not computers.

The shame in all this is that traditional mathematics pushes for an ideal of comprehension that nobody lives up to, not even first-graders, whose brains are deemed so ill-developed that they couldn't possibly understand mathematics without the crutch of intuition. I think just one example will suffice to show this. Most first-graders learn how to add 777 and 123 using the addition algorithm, many learn how to do this with ease. And I am sure that not a single first grader ever conceives of apples or anything else when they apply the addition algorithm to these numbers. They follow strict rules and become symbol manipulators, they become —in a very real sense— computers. This cannot be denied! Five year old children are capable of manipulating symbols in a very, very abstract way, divorced entirely from "real-world" meaning.

This came up the other day when I was talking with an old friend —now a fifth-grade teacher— on the beach about mathematics pedagogy. I explained to him some aspects of the calculational style, and he found them thrilling. (Although it may have just been the wind.) His only objection was that the level of abstraction used in the calculational style was inappropriate for fifth graders; little kids just wouldn't get it. However, when I pointed out that they were already working at an abstract level when they used the addition algorithm, he realized immediately that it had been ridiculous to think that children needed a pictorial crutch, when clearly they worked fine without it.

I thought it had been a revelation, yet later on he told me that he still felt that young kids needed conceptual explanations of mathematical ideas; in his mind abstraction was very difficult to grasp. But because of the addition algorithm example, because I knew that young kids were capable of abstract thought, I was left wondering whether this difficulty existed only because traditional mathematics education places such emphasis on conceptualization. And since the students of today become the teachers of tomorrow, it is no surprise that this mental disability is passed down from generation to generation.

Aside. There is another area of mathematics where most students become excellent calculators: basic algebra. When faced with an equation like $2 * x + 1 = 3$, no student tries to understand it intuitively as "two times the quantity I'm looking for plus one is equal to three" ; they simply subtract 1 from both sides of the equation to yield $2 * x = 2$, and divide both sides by 2 to yield $x = 1$. I'm not sure the abstract concepts involved in solving an algebraic equation are any more difficult than those involved in applying the arithmetic algorithms —in fact, they might be much easier— .

We may legitimately raise the question, though, of why calculational methods have only gained wide acceptance in basic arithmetic and algebra. The best answer I can come up with is that, until very recently, a calculational methodology was all but nonexistent in other fields. (End of Aside.)

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