Trip report: Lessons learned? (at Calculus Camp)

One student had no idea how to differentiate:

$$x \ln^3 x$$

until I had her write it as:

$$x * (\ln x)^3 \qquad .$$

Then it was clear that the expression was a product, so the product rule was needed. Similarly a student had difficulty simplifying:

$$1 \;=\; \sin(x+y)(1+y')$$

by solving for y' . Until I wrote it as:

$$1 \;=\; (\sin(x+y)) * (1+y') \qquad .$$

Woe to invisible multiplication!

But perhaps more importantly, what concerned me is that the students working on these problems didn't understand what had gone wrong: they were confused as to the source of their confusion. They are pretty good calculators when it comes to algebra, but they have never been given any appreciation for syntax. My future students will be taught to write carefully, and respect the syntax that makes calculation possible!

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<u>Addendum</u>

The heterogeneity of function application notation is a shame:

$$\sin x \quad , \quad \sin^2 x \quad , \quad \log_b x \quad , \quad f(x) \quad , \quad x^6 \quad , \quad \text{etc.}$$

It is not a wonder students get confused. As much as possible, I prefer $f.x$: The low dot indicates function application; the thing to the left is the function; the thing to the right is the argument.

This all came up because students were supposed to recognize, for instance:

$$\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h}$$

as the definition of the derivative of $\sqrt[3]{x}$ at $x = 8$. This process involves:

- recognizing the limit as a derivative

which involves

- recognizing the function (here $\sqrt[3]{\ }$)
- recognizing the point of evaluation (here 8 , notice that $2 = \sqrt[3]{8}$)
- taking the derivative of the function
- evaluating it at the point of evaluation .

Because of the aforementioned heterogeneity of notations for function application, many students have trouble in the first three steps.

Note though that these limits are always of the form:

$$\lim_{h \to 0} \frac{f.(x+h) - f.x}{h} \quad ,$$

which satisfies the condition for L'Hopital's rule. (For the uninitiated: both numerator and denominator should equal 0 at the limit point.) To solve such problems using L'Hopital's rule, one must:

- recognize that L'Hopital's rule is applicable
- differentiate the numerator
- differentiate the denominator
- "plug in" $h = 0$.

These steps are much simpler than the others above, and more general. L'Hopital can be applied in many instances, not just in such problems. And to recognize its applicability, one need only evaluate functional expressions at a point, whereas with the previous strategy, one has to seek out hidden functions. Differentiating the numerator amounts exactly to differentiating the hidden function, though thanks to the power of the derivative, one need not find the hidden function in order to differentiate an expression containing it. Differentiating the denominator, h , is a walkover. So is plugging in $h = 0$.

So why do teachers recommend the former method over the latter, especially when they see how error-prone the former method can be? I dare not venture a guess.

A final note

Above I said: "Thanks to the power of the derivative, one need not find the hidden function in order to differentiate an expression containing it." . This is a testament to the beauty of the derivative, and also sheds some light on why integration is so much harder than differentiation: To differentiate, we only need to apply a small handful of rules, which in combination allow us to take the derivative of almost any function imaginable. On the other hand, we only know a few forms we can integrate, so integration consists of rewriting the integrand in one of these forms. Sometimes this rewriting requires a good deal of ingenuity and luck; often it is impossible.

The point can be made more generally: Whenever we can solve a task either by "turning the crank" or by ingenuity, always turn the crank. We can only make the wrong choice if we have a choice to make.

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