

Why math is frightening

This note is an application of JAW79 , but is self-contained.

For many, math is confusing and frightening. Naturally there are many reasons why this is the case, but the biggest is math's sheer simplicity.

We should appreciate the mind's ability to handle fuzzy, complex concepts with relative ease. Imagine a red apple, for instance. This concept gives us no trouble, even though red apples have more properties than we could ever hope to list. For instance, they are red, crunchy, sweet, small enough to fit in a hand, they have seeds, they grow on trees, they are a part of nature, they have a weird apple-shape, they have some sort of leafy navel on their underside, they can be purchased in supermarkets, they can be eaten, etc ad nauseum. What a list!

But I would like you to consider that it is the scope of this list which is responsible for our natural ease with the concept of a red apple. It is tied in securely to our network of psychological concepts. And moreover, the concepts it is linked to are very familiar. Most things we know have color, taste, etc, even if we can only imagine these properties.

Now consider one of my favorite concepts, a *blorg* . A blorg is an abstraction of a red apple, a severe abstraction in fact. Blogs have only one property: they are red. It would be very difficult to argue that blogs are more complex than red apples. How can they be, when blogs only have one property, while red apples have more than we could ever list?

So blogs are simpler, but they are ever more disconcerting, because they do not fit in securely to that network of concepts mentioned above. There is nothing in our psychological experience like a blorg: nothing is only red. So when something like a blorg is mentioned, we try very naturally to create the missing ties to our conceptual network. We ask questions: *How big is a blorg? What is it used for? Would I be afraid to run into one in a dark alley? Can I eat it, like an apple?* (Notice the use of analogy in such questions.) However, none of these questions can be answered, and for many, this state of affairs causes discomfort.

These questions cannot be answered, because they have no answers. They are completely **irrelevant** questions. It makes no sense to ask how big a blorg is, because blogs only have a property of color. Nothing about blogs would ever allow you to answer that question.

Mathematical concepts are like blogs, in their utter austerity. In order to become comfortable with mathematical concepts, we have to learn to let go of our natural inclination to ask irrelevant questions about them.

Recently, I have taken to asking people: "What is 3 ?" . Every single person I have asked has given the same answer: "A number." . And they are right: 3 is a number. But oh, it is so much more than a number. For example, 3 is less than 4 , and greater than 2 . It is odd, prime, and the smallest odd prime. It is equal to $1 + 2$. It is the square root of 9 , and the square of no natural number. And so on. It is a veritable red apple! No wonder so few people have problems with the concept of 3 .

On the other hand, people have a lot of problems with the notion of a variable. But

what could be simpler? Similar to the way we made a blog from a red apple, let's abstract away from the number 3, to form a new concept. Since I have to give it a name, I'll call it x , and like the blog, it will only have one property: it is a number. Now we really do have a concept that is "just a number" !

And there, ladies and gentlemen, is a variable. A variable is a concept with only one property: "the kind of thing it is" . (That property is called the variable's **type** .) Usually, in elementary mathematics, variables have the property of being a real number, and nothing else. That's it. Is x less than 3 ? Who knows. Equal to 3 ? Who knows. Greater than 3 ? Who knows. But I know that x is either less than, equal to, or greater than 3, because any number has this property! Moreover, because I can add things that are numbers, I am welcome to describe the number $x + 1$. What is this number? Who knows, but at least I know that it is greater than x ! And finally, suppose I am creating a mathematical model of a situation where I am trying to find out the age of a farmer. Because an age is a number, I am welcome to name that age x . (Then x has two properties: it is a number, and is equal to the age of the farmer in question.) What could be simpler, or more elegant, than a variable?

Note how many educators try to hide variables from students, with the caveat that they're "too complex and abstract" for students to understand. They may be frightening, because they evoke irrelevant and unanswerable questions, but shouldn't we be teaching students to rid themselves of such irrational fears, rather than keeping them afraid?

And open up a math textbook to the page where they introduce variables. I guarantee you will see the authors of the textbook trying, in vain, to answer some of these irrelevant questions!

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