A high-tech calculation

As a member of a High-Tech University, I sometimes feel obliged to exhibit fantastic calculations. Here is one.

When we have a tedious calculation of the form

$$
\begin{array}{ll}
 & [x \equiv y] \\
 & \vdots \\
\Rightarrow & \vdots \\
 & [P.x.y]
\end{array}
$$

there is no need to redo all the work in order to strengthen the last line with conjunct $[P.y.x]$. Here we exploit symmetry, and simply continue our calculation with

$$
\begin{array}{ll}
= & \{\text{symmetry}\} \\
 & [P.x.y] \wedge [P.y.x] \, .
\end{array}
$$

The simplest application of the above principle is

$$
\begin{array}{lll}
 & [x \equiv y] & (0) \\
\Rightarrow & \{\text{pred. calc.}\} & \\
 & [x \Rightarrow y] & (1) \\
= & \{\text{symmetry}\} & \\
 & [x \Rightarrow y] \wedge [y \Rightarrow x] & \\
= & \{\text{pred. calc.}\} & \\
 & [x \equiv y] & (2)
\end{array}
$$

Comparing lines (0), (1), and (2) we have shown

$$[x \equiv y] \equiv [x \Rightarrow y] \, .$$

That's what we call High Technology.

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