A note on equation $x : [\, t \vee s;x \equiv x \,]$

We consider the "homogeneous" linear equation

(0) $\quad x : [\, s;x \equiv x \,]$,

which – by definition – has ∞s as its weakest solution.

We also consider the "inhomogeneous" linear equation

(1) $\quad x : [\, t \vee s;x \equiv x \,]$.

Now we have the following theorem.

<u>Theorem</u> The weakest solution of (1) is $r \vee \infty s$, where r is any solution of (1).

<u>Proof</u>

<u>$r \vee \infty s$ solves (1)</u> :

$$
\begin{array}{ll}
 & t \vee s;(r \vee \infty s) \\
= & \quad \{\text{r.c.}\} \\
 & t \vee s;r \vee s;\infty s \\
= & \quad \{ r \text{ solves } (1) \} \\
 & r \vee s;\infty s \\
= & \quad \{ \infty s \text{ solves } (0) \} \\
 & r \vee \infty s \quad .
\end{array}
$$

<u>$r \vee \infty s$ is the weak extreme of (1)</u> :

We show that for any x

$$[\, x \Rightarrow r \vee \infty s \,] \Leftarrow [\, t \vee s;x \equiv x \,] \quad :$$

$$
\begin{array}{ll}
 & [\, x \Rightarrow r \vee \infty s \,] \\
= & \quad \{\text{p.c.}\} \\
 & [\, x \wedge \neg r \Rightarrow \infty s \,] \\
\Leftarrow & \quad \{\ \infty s \text{ is the weak extreme of } (0)\ \}\{\text{KT}\} \\
 & [\, s;(x \wedge \neg r) \Leftarrow x \wedge \neg r \,] \\
= & \quad \{\text{p.c.}\} \\
 & [\, r \vee s;(x \wedge \neg r) \Leftarrow x \,] \\
= & \quad \{\ r \text{ solves } (1)\ \} \\
 & [\, t \vee s;r \vee s;(x \wedge \neg r) \Leftarrow x \,] \\
= & \quad \{\text{r.c. : this is a macro}\} \\
 & [\, t \vee s;r \vee s;x \Leftarrow x \,] \\
\Leftarrow & \quad \{\text{p.c.}\} \\
 & [\, t \vee s.x \equiv x \,] \ .
\end{array}
$$

(<u>End</u> of Proof.)

The analogy of the above with ordinary differential equations is striking – hence the ad-hoc jargon – . I did not know the theorem – hence this note – . The above result has been triggered directly by Rutger-5, which uses a special instance of the theorem (Rutger Dijkstra may have been aware of the theorem.)

The question that remains is whether we should explore, in more depth, equations à la the above and their properties. I think we should, if only for fun.

WHJ Feijen
Sterksel, 9 August 1993

(Written with MB Meisterstück.)